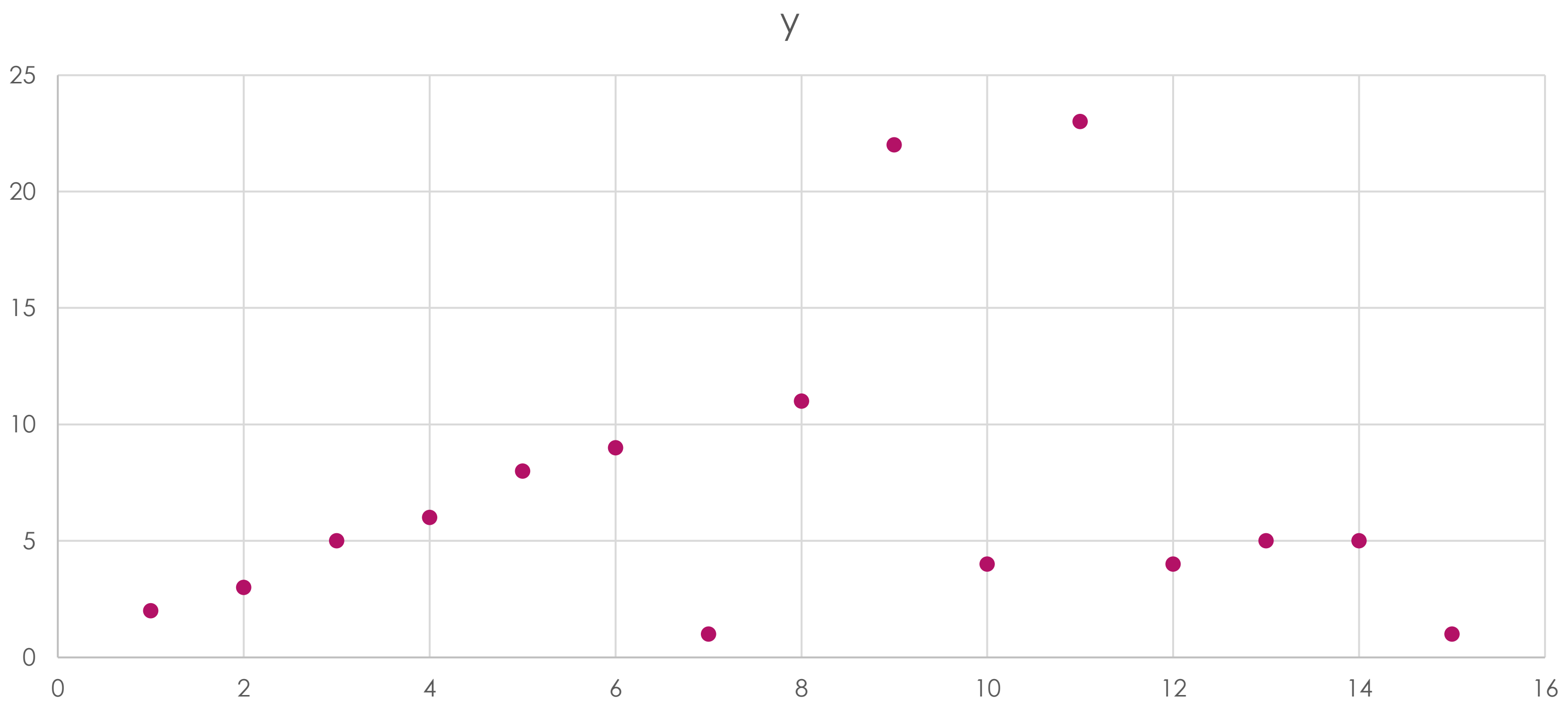
Linear Least Square Problem (LSP) and the Bias-Variance Trade-off in Machine Learning

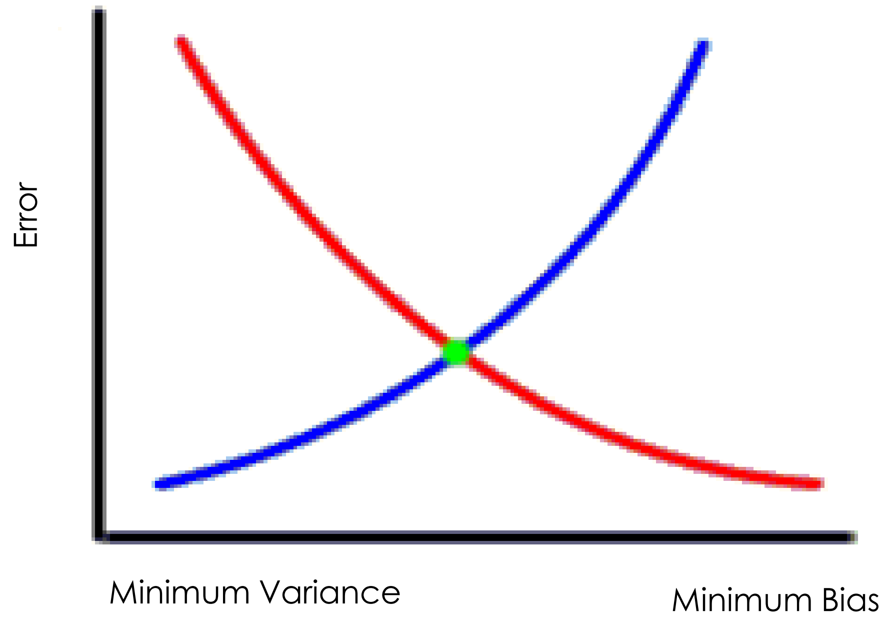
MODERATOR: Prof Montaz Ali

Group Members:
Fridah Meso
Atlegang Molubi
Ntombifuthi Khumalo
Ayobami Akinyelu
Valentine Azom
Africa Mgudlwa
Ndivhuwo Ndou
Rudzani Nemusunda
Edward Nonyane
Salma Omer
Ebenezer Popoola

Introduction



Bias Vs Variance



Simplified Linear Regression Model

- Generate General Model from Big Data
- Initially, Model is unknown
- Due to lack of dataset, we are currently trying to generate a dataset using a known function
- Procedures include

Procedures

- Generate Intervals [-2, 5]
- F(y) = $y^4 - 3y^3 + 2y^2 + 5y + 2$
- Generate $b_i y_i$ for 500 points (b_i= f(y_i) + ε)
- Randomly select 3 groups of 50 points from the 500 points
- Solve Ax = b to derive h_i for each group
- Find bias and variance
- Go through the same process for quadratic, cubic and others

Squared Residual of the LSP

- Variance = $\frac{1}{n}\sum_{i=1}^{n}(h_i - \bar{h})^2$
- Bias = $(F(y_1) - \bar{h})^2 + (F(y_2) - \bar{h})^2 + \ \dots + \left(\left(F(y_n) - \bar{h}\right)^2\right)$

Squared Residual of the LSP

$E(x) = \sum_{i=1}^{k} x_i f(x_i) = \sum x_i P_r(X = x_i)$ --- (1)

$\int xf(x)dx = E(x) = \mu$ --- (2)

$Var(x) = \int (x - \mu)^2 f(x)dx$ --- (3)

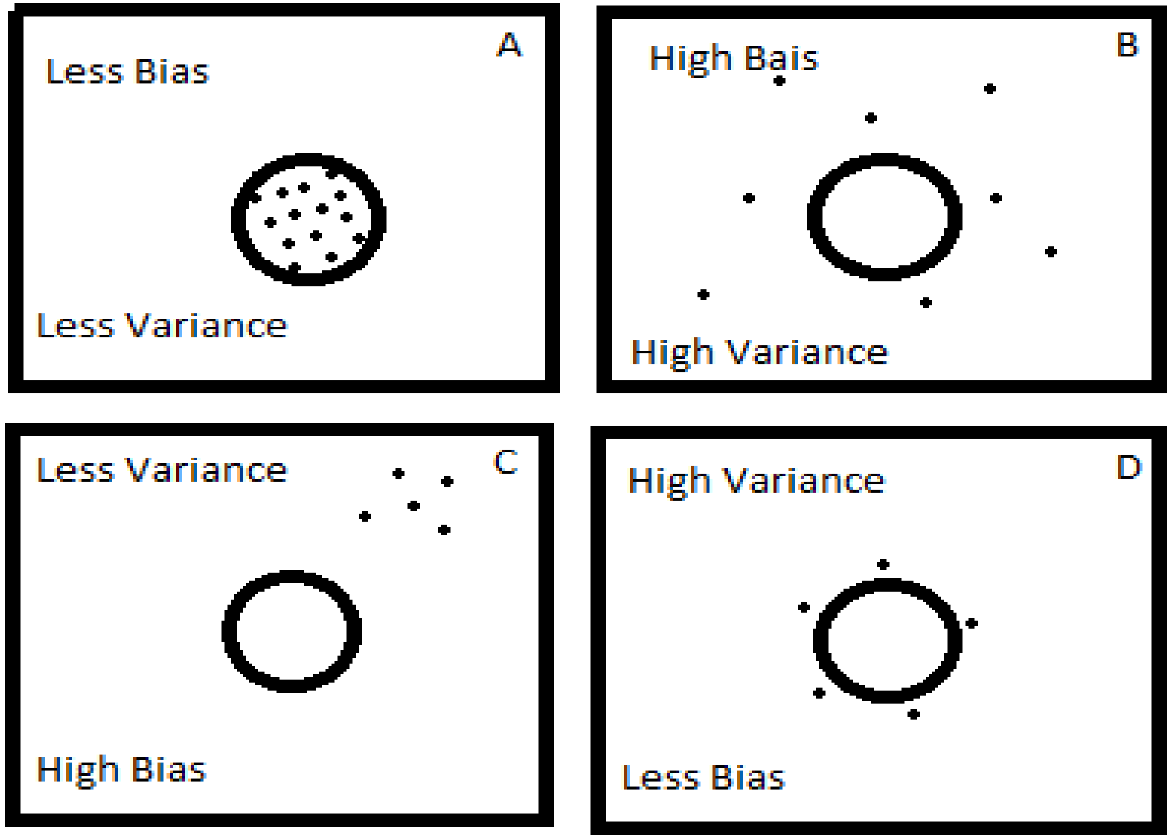
$E[x^2] = Var[x] + (E[x])^2$ -- (4)

Evaluating the expectation between b and h (i.e. the model):

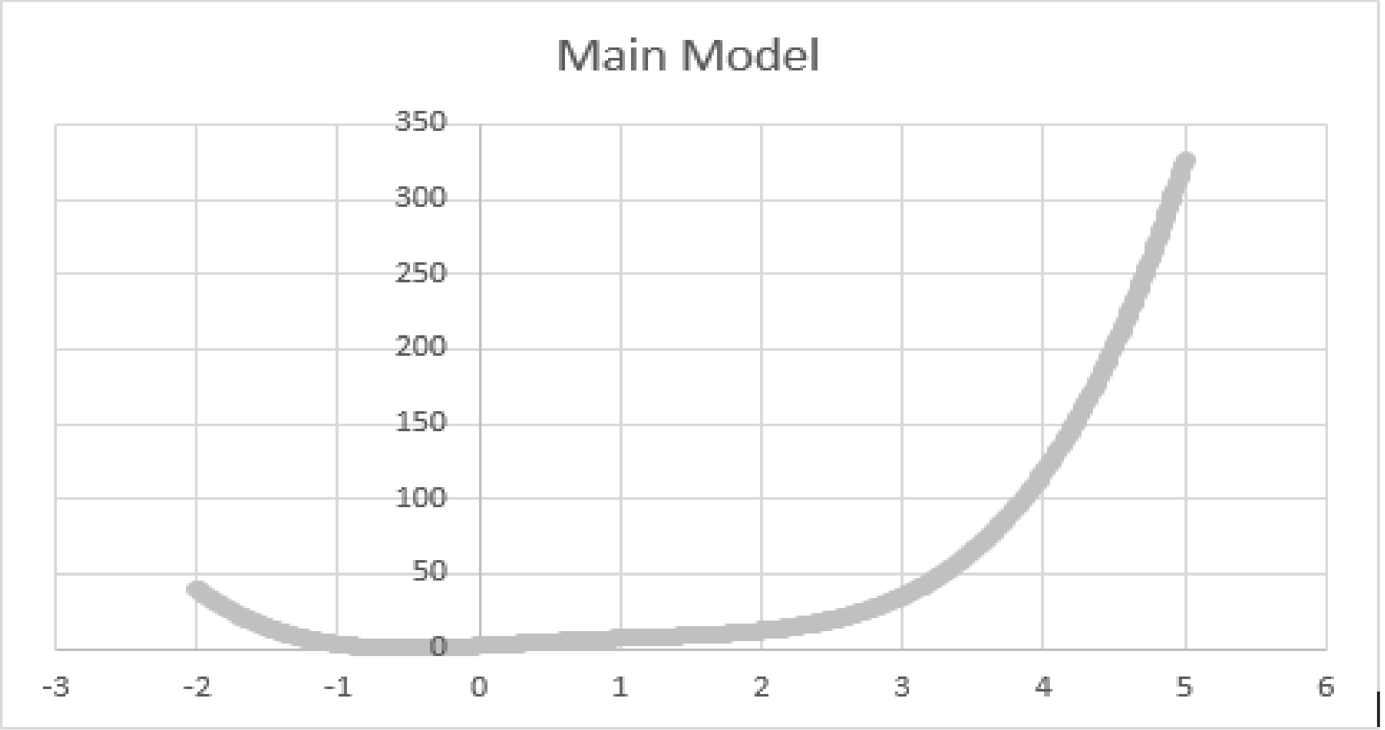
$E\left[\left(b - h(y)\right)^2\right] = E[b^2 - 2bh(y) + h(y)^2]$ --------------------------------------- (5)

$= \sigma^2 + E\left[(h(y) - E[h(y)])^2\right] + (f - E[h(y)])^2$ --------------- (6)

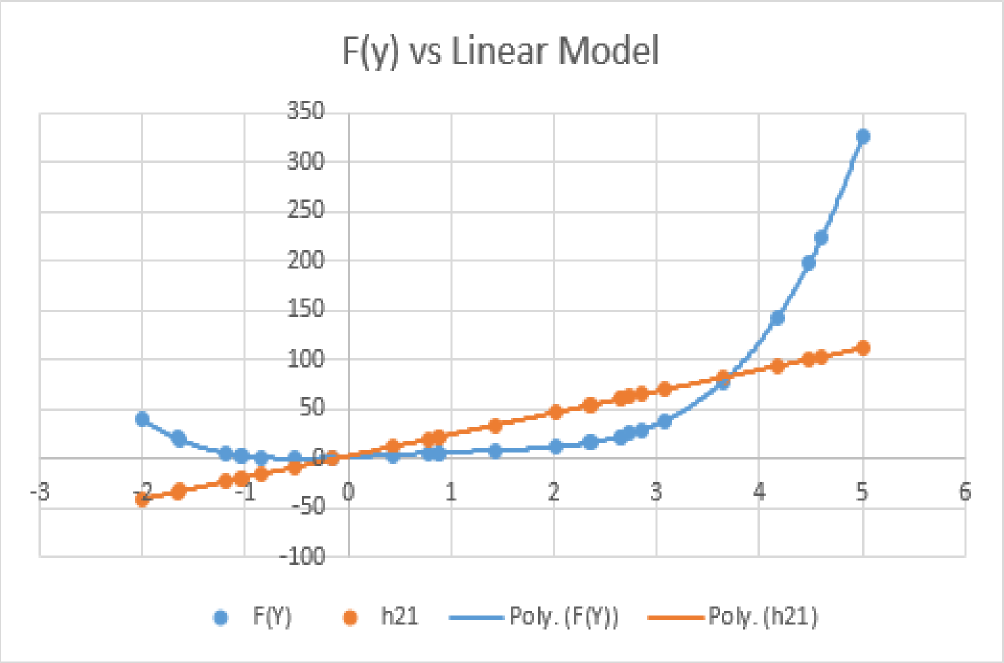
Bias vs Variance

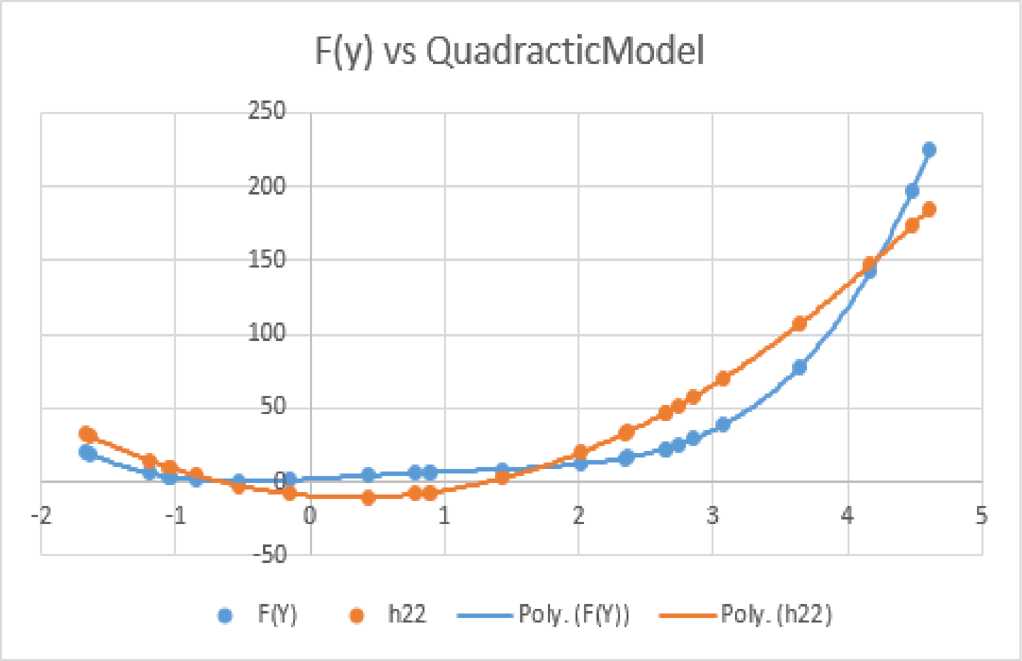


Result

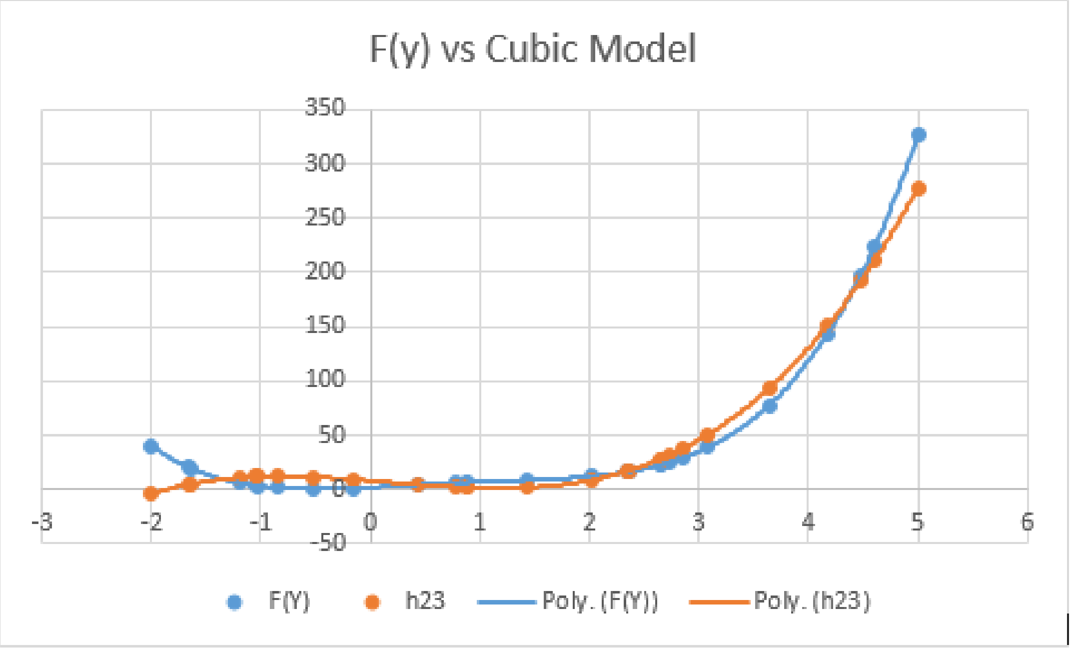


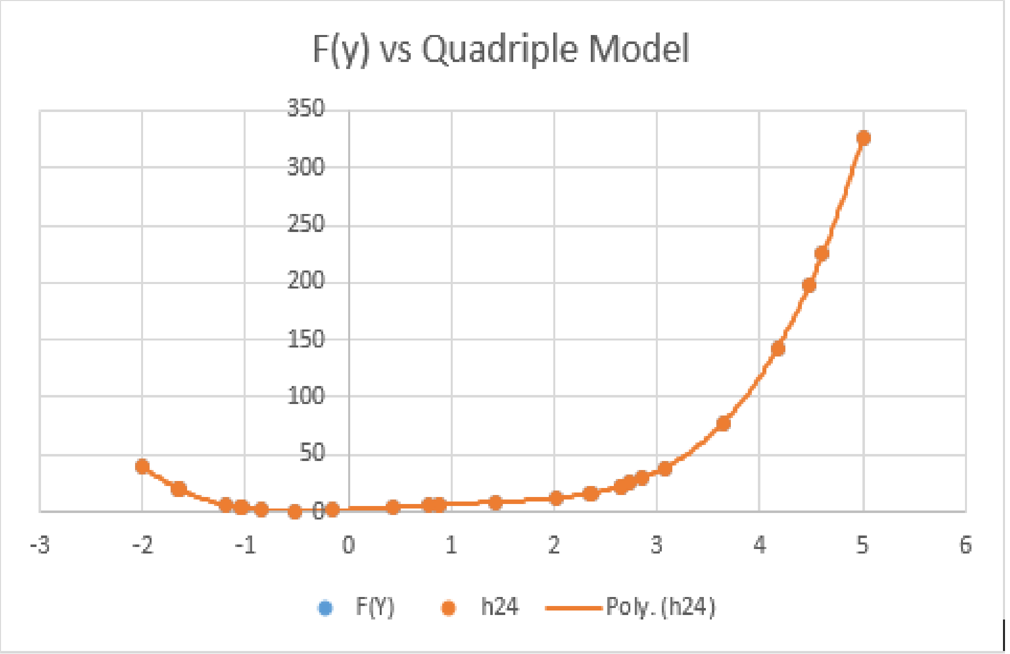
Result (Cont'd)



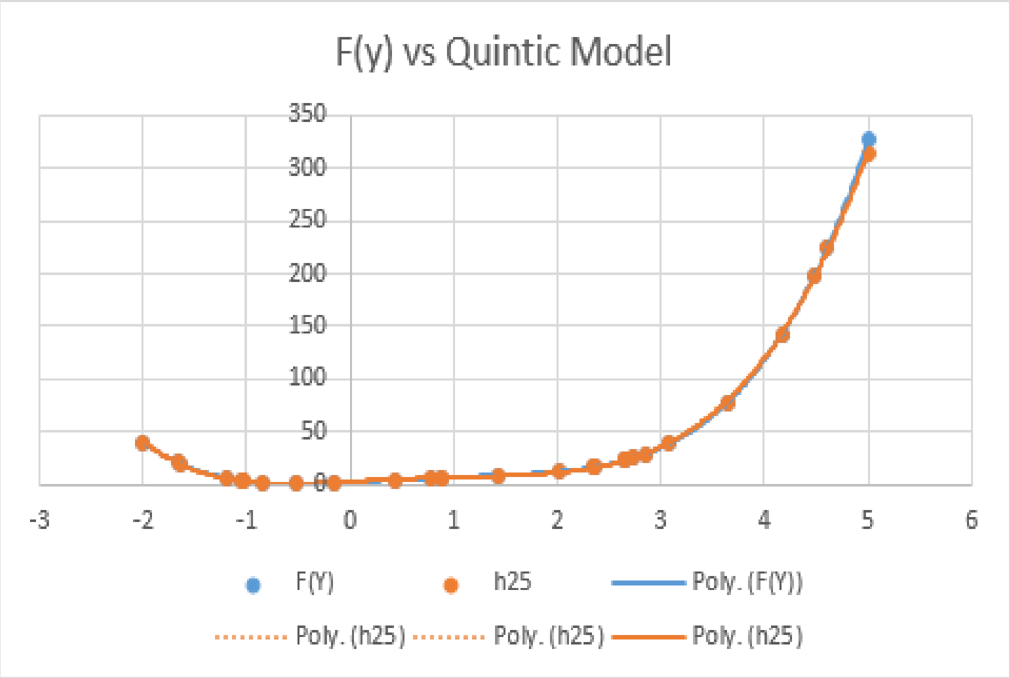


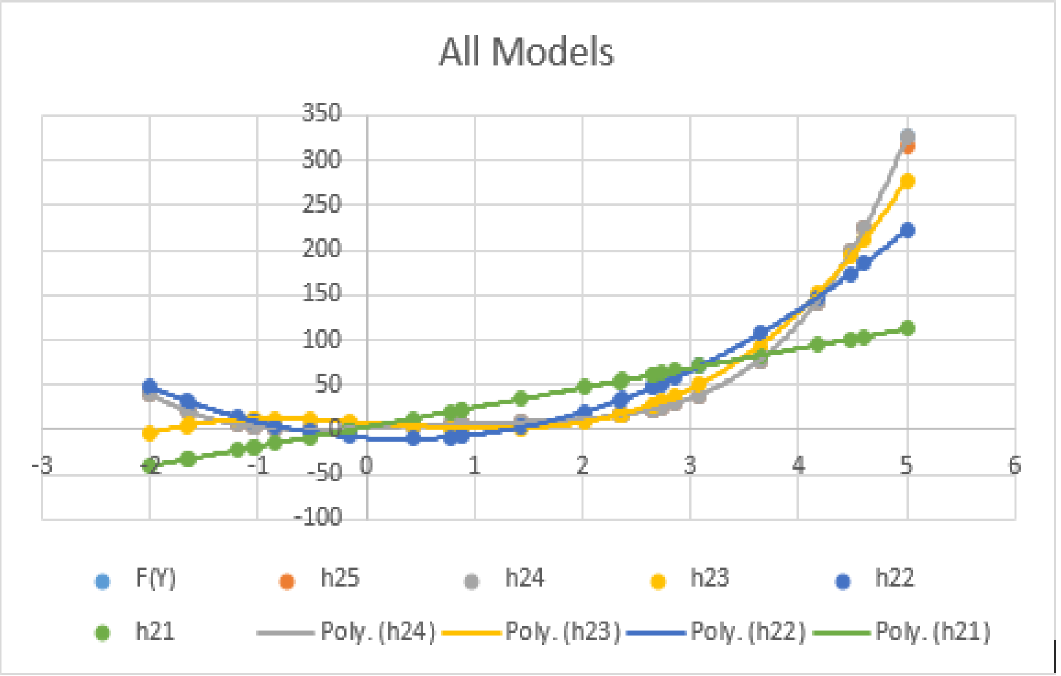
Result (Cont'd)





Result (Cont'd)





THANK YOU!